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Potential GDP Estimation for Romania[*]

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Abstract

The paper's objective is to estimate the growth rates of potential GDP for Romania. To this end, we present and implement several univariate and multivariate methods for the measurement of potential GDP growth: production function, filters with unobservable components, structural vector autoregressions. The results are robust to changing approaches and specifications, pointing to an acceleration in the annual growth rate of potential GDP from an average of 3-4 percent between 2000-2002 to values around 6 percent in the recent period.

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Note

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INTRODUCTION

Economic growth displays irregular fluctuations along business cycles, periods of economic expansion alternating with those of stagnation. The fact that economies have a central long-term growth tendency has prompted economists to formulate the concept of potential GDP, whose long-term growth rate is the result of fundamental factors: the structure of the economy, its productive capacity determined by technology, demographic and educational factors which influence the labor force, and others. This concept corresponds to the steady-state growth rate featured in neoclassical models and it generally changes over relatively long time spans only as a result of significant and lasting changes in economic fundamentals.

In order to be empirically identifiable, the concept of potential GDP has received various definitions in literature (for a presentation of alternative or complementary definitions see Benk, Jakab and Vadas, 2005). In this paper we define potential GDP as the level of real GDP which the economy can produce without generating inflationary pressures. In this framework, the medium-term growth rate may vary, temporarily deviating from its long-run equilibrium value. The GDP gap (excess demand) is the difference, expressed in percentage points, between actual real GDP and potential GDP, representing a synthetic aggregate indicator of inflationary pressures. Consequently, excess demand is a variable that carries a lot of relevance for establishing the monetary policy stance, which is aimed at bringing inflation to the announced target over the relevant horizon by managing aggregate demand.

This paper presents a series of univariate and multivariate methods for determining the growth rate of Romania's potential GDP: production functions, filters with unobserved components, structural vector autoregressions. The results are robust to different specifications and methods, indicating an accelerating annual potential GDP growth rate from an average of 3-4 percent in 2000-2002 to values of around 6 percent in more recent years.

The paper is structured as follows: section one briefly surveys the existing estimates of Romania's potential GDP from other empirical studies, section two details the methods used to determine the potential GDP and the resulting estimates, while section three concludes.

1. EMPIRICAL STUDIES ESTIMATING ROMANIA'S POTENTIAL GDP

In contrast to other Central and Eastern European countries for which there are both panel and individual country studies, empirical results regarding the evolution of potential GDP in Romania are scarce. One possible cause is the relatively short time span covered by the relevant data.

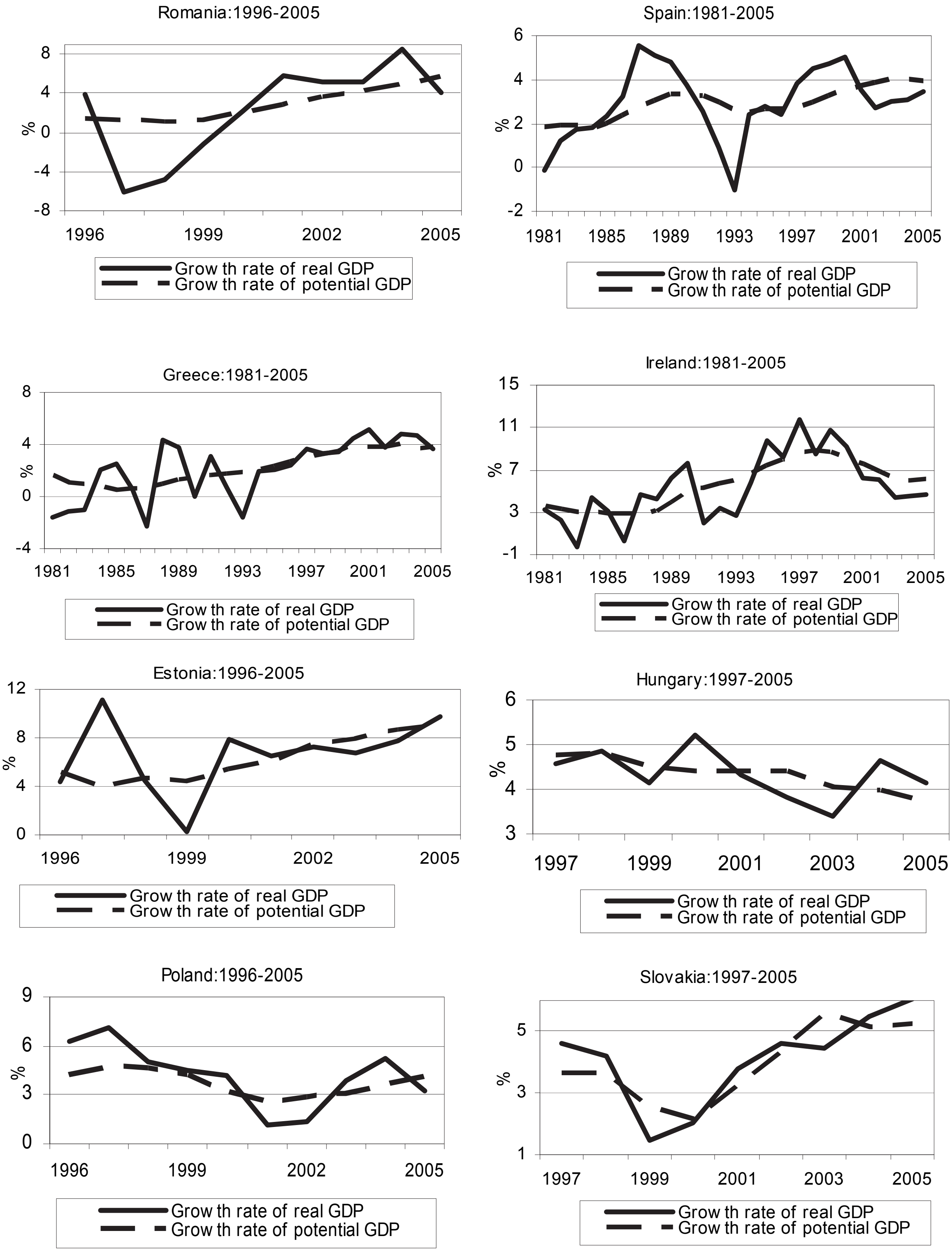
Denis et al. (2006)[1] are the first to include a detailed assessment of Romania's potential GDP. Their estimates for EU27 are made within a comparable framework using the production function approach. The growth rates of real and potential GDP for 8 member countries, including Romania, are shown for comparison in Figure 1. In Romania's case, the authors point to an average annual growth rate of potential GDP for the 1996-2006 period of 3.2 percent. The last years display higher values, the average for 2003-2006 being 5.6 percent.

Romania's long-term GDP growth rate is also mentioned in the IMF Report on Article IV Consultation with Romania, IMF (2006a). In the original model, IMF (2006b), on the basis of which the results were obtained, the derivation of the efficient level of production implies an optimal utilization of resources and existing technologies. The sample used for the panel estimates includes 145 countries. Total factor productivity growth, one of the main sources of potential GDP growth, reflects differences in the efficiency of using the factors, and not differences stemming from access to technology, which is assumed equal for all countries under review. In the case of Romania, the projected paths of economic growth in the next decade are based on three different scenarios for the authorities' capacity to absorb EU funds and successfully implement an institutional and structural reform program. The baseline scenario is conservative, in the sense that authorities do not fully implement the reform program, and public investments proceed at a low rate. This scenario yields a sustainable long-term growth rate of real GDP of 5 percent on average for the period 2006-2015. In the optimistic scenario the sustainable economic growth rate is 7.5 percent for the same period, while in the pessimistic one it drops to 3 percent.

Dobrescu (2004) identifies the output gap variations for the period between 1991Q1 – 2002Q4 by completing the previously-mentioned definition with an additional constraint. Thus, potential GDP is also linked to the existence of a constant share of net exports in GDP. The evolution of potential GDP thus defined shows wide annual variations.

[1] The paper was initially published during 2006 without any reference to Romania. When updating the results, the authors included for the first time in their sample data relative to Romania and Bulgaria. These estimates, which are also included in the European Commission forecast report (2006), are available at: http://forum.europa.eu.int/Public/irc/ecfin/outgaps/library?1=/2006_autumn_forecast/autumn_2006 results/autumn_2006 principal&vm=detailed&sb=Title.

Figure 1. Growth rates of actual and potential real GDP for eight countries



Source: Denis et al. (2006)

2. ESTIMATING POTENTIAL GDP

Given that potential GDP is a variable which cannot be directly observed from statistical data (unobservable), the task of measuring it is not a trivial exercise. The methods used for estimating the potential GDP can be divided into univariate methods, which only analyze the evolution of real GDP, and multivariate methods, which simultaneously take into account several macroeconomic variables. Each of these methods comes with advantages and disadvantages.

Because of the fact that the level of potential GDP is not directly observable from statistical data, there are several problems common to all the methods employed in its estimation. One of them is the very short sample of usable data for Romania, which probably does not cover an entire business cycle. Due to the frequent structural changes which took place in the economy in the period under scrutiny, it is possible that, regardless of the chosen method, the identified trends do not correctly reflect these changes. Another issue is the fact that official statistical GDP data is published with a lag, being subsequently subjected to revisions. These circumstances increase the uncertainty of real-time estimates for potential GDP and excess demand, requiring revisions as new information becomes available.

These aspects justify the application of alternative methods to identify the evolution of potential GDP in this paper. This approach allows an assessment of the robustness of results, given that each separate estimate has a relatively high degree of uncertainty. However, it must be noted that the methods used do not correspond strictly to the specific definition chosen for potential GDP in the paper.

In this paper we have used statistical data for the 1998Q1 – 2006Q2 period. The data sources are publications of the National Institute of Statistics, of the National Bank of Romania and data provided by the General Administration of Customs, the European Central Bank and the US Bureau of Labor Statistics. Details regarding the source of each time series, as well as the transformations applied to the data can be found in Appendix 1.

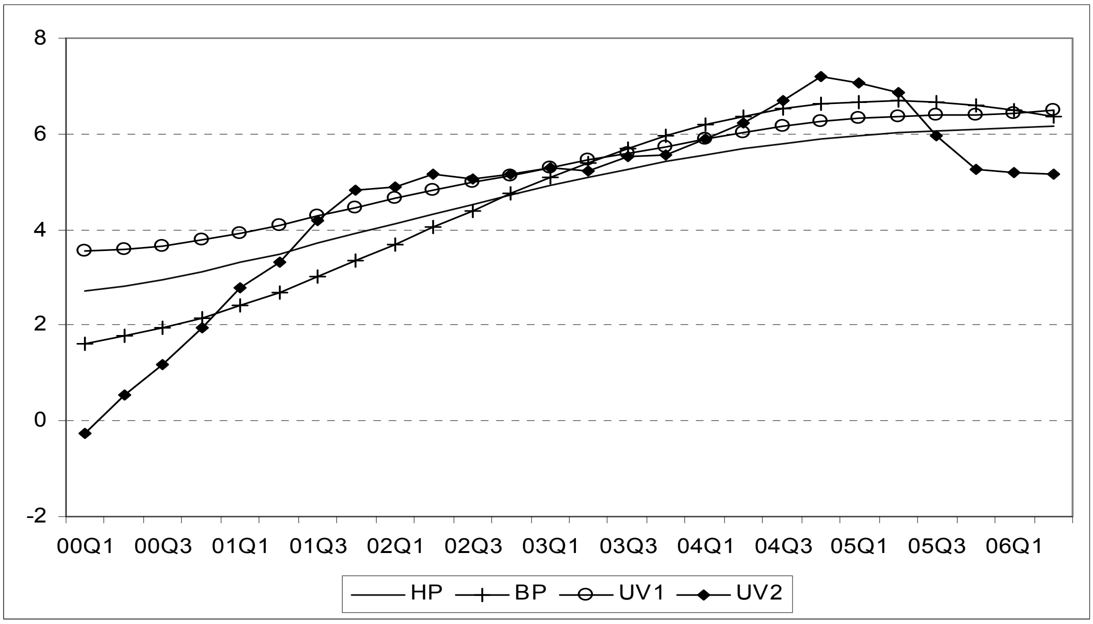
2.1. Univariate methods

Univariate methods decompose real GDP into potential GDP and excess demand by way of a purely statistical analysis of the past behavior of the time series, based on a set of assumptions regarding the dynamics of the trend and the gap. The advantages of univariate methods reside in the fact that they do not rely on the implications of any particular economic theory (so the relevance of the results does not depend on the assumptions of the theory) and the fact that they can be easily implemented. The disadvantages are a direct consequence of their univariate character, the possible interactions with other macroeconomic variables thus being disregarded.

In addition, the output of these methods does not strictly correspond to a precise economic definition of potential GDP or of excess demand.

The univariate methods used in this paper for measuring the growth rate of potential GDP are the Hodrick-Prescott filter, the band-pass filter and two univariate models with unobserved components based on the Kalman filter[2]. Averaging the results, these methods indicate an acceleration of the average annual growth rate of potential GDP from 3.6 percent in the 2000Q1 – 2002Q4 interval to 6 percent in the 2003Q1 – 2006Q2 interval, with values varying in a range between 5.7 – 6.2 percent in the latter period (Table 1).

Figure 2. Annual growth rates of potential GDP – univariate filters (%)



2.2. Multivariate methods

When decomposing real GDP, multivariate methods also use information contained in other macroeconomic variables, taking into account the cross-dependencies between these and real GDP. The most frequently used multivariate methods of estimating potential GDP are production functions, multivariate unobserved components models and structural vector autoregression models. The main advantage of these methods is that they benefit from a larger information set, information which is then interpreted through the relations between variables suggested by the economic theory. The main disadvantage is that they require longer time-series than univariate methods.

[2] General aspects regarding the Kalman filter can be found in Appendix 2, and specific details regarding univariate filters in Appendix 3.

2.2.1. The production function approach

The production function approach has the advantage of reflecting the supply side of the economy. The main drawback is the restrictiveness of the assumptions regarding the functional form and the utilization of production factors.

The method was applied using quarterly data covering the 1998Q1 – 2006Q2 period for the labor force[3], GDP and gross fixed capital formation (the latter two in real terms). Given the short time span for which data is available and the lack of other necessary parameters, the Cobb-Douglas production function was chosen as a functional form. The capital and labor weights were assumed to be 0.33 and 0.67[4], respectively.

Computation of the capital stock is the main challenge of this method. Given the very large variations in the annual fixed capital stock[5], caused by including frequent reevaluations in the series provided by the National Institute of Statistics, the use of this series would seriously damage the relevance of the estimates. The lack of an adequate data series for this indicator also precludes the use of gross fixed capital formation for determining a variable depreciation rate. Taking all of these into account, the chosen annual depreciation rate was similar to the one generally used in the literature, namely 5 percent (implying a quarterly rate of 1.23 percent).

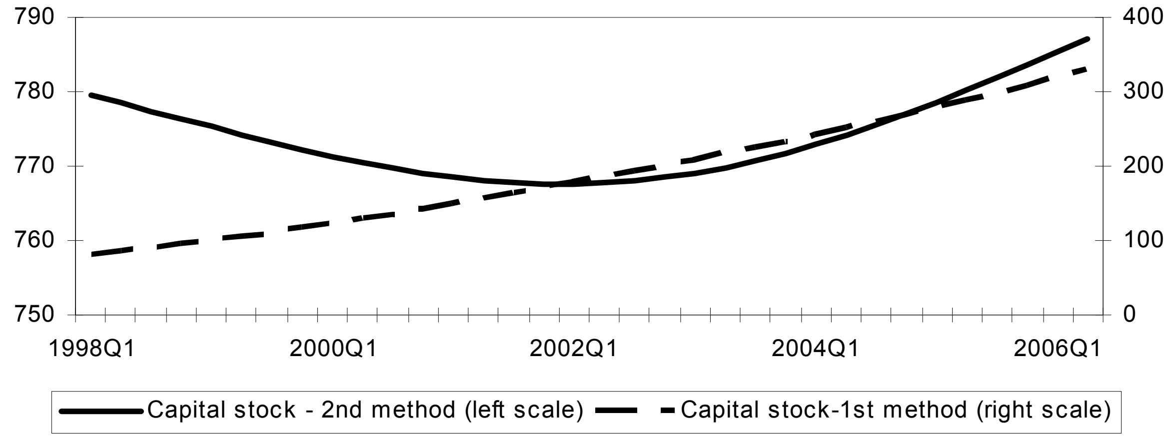
To overcome these constraints, the solution was to determine first the initial capital stock and subsequently, using the accumulation equation presented below and the data on gross fixed capital formation and the depreciation rate, to iteratively generate the capital stock series. Two methods were used for determining the initial capital stock:

- the first one, similar to the approach of Bergoeing et al. (2002) for annual data, has assumed that the ratio of the capital stock values in the first two periods is equal to the geometric average of the ratios in the first twelve quarters[6].

[3] The average number of hours worked per day is assumed to be 8.
[4] For details, see Appendix 4.
[5] For example, the ratio of the fixed capital stock at the end of 2003 and the one existing at year-end 2002 is 2.35, mainly due to the evolution of the majority state-owned capital stock (NIS, "Romania's Statistical Yearbook: 2005").
[6] $\frac{K_{T+1}}{K_T} = (\frac{K_{T+12}}{K_T})^{1/12}$

– the second one, similar to the approach of Harberger (1978), assumes a capital growth rate equal to the average growth rate of real GDP for the analyzed period[7].

Figure 3. The capital stock (RON billion, 2003 constant prices, seasonally adjusted)



The two resulting capital stock series are presented in Figure 3. It has to be pointed out that the two methods have generated different dynamics and levels of the capital stock. Once the capital stock is determined, total factor productivity was computed as a residual of the production function; the level of potential GDP was arrived at by aggregating the trends of the components thus identified.

The growth rates computed from the level of potential GDP thus measured display increasing dynamics, reaching almost 6 percent at the end of the sample, as can be seen from Figure 4. Looking at the 2003Q1-2006Q2 period, the results indicate an annual average growth rate of potential GDP of 5.7 percent for the first approach, and 6.0 percent for the second, while the growth rates for the entire sample are lower (Table 1).

2.2.2. Multivariate filters with unobserved components

The measurement of potential GDP using multivariate filters with unobserved components[8] makes use of the information contained in other macroeconomic variables, interpreted through the relations among the variables implied by the economic theory. In order to decompose the real GDP series, these models generally use the interactions with the real interest rate and the real exchange rate (through the aggregate demand curve equation), with the unemployment rate (through "Okun's law") and with the inflation rate (through the Phillips curve equation).

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[7] The capital accumulation equation can be rewritten as: $\frac{K_{t+1}-K_t}{K_t} = -\delta + \frac{I_t}{K_t}$, where δ is the depreciation rate. Assuming that the capital growth rate is equal to the average growth rate of real GDP (g in the equation), we have the following formula for determining the initial capital stock: $K_t = \frac{I_t}{(g+\delta)}$

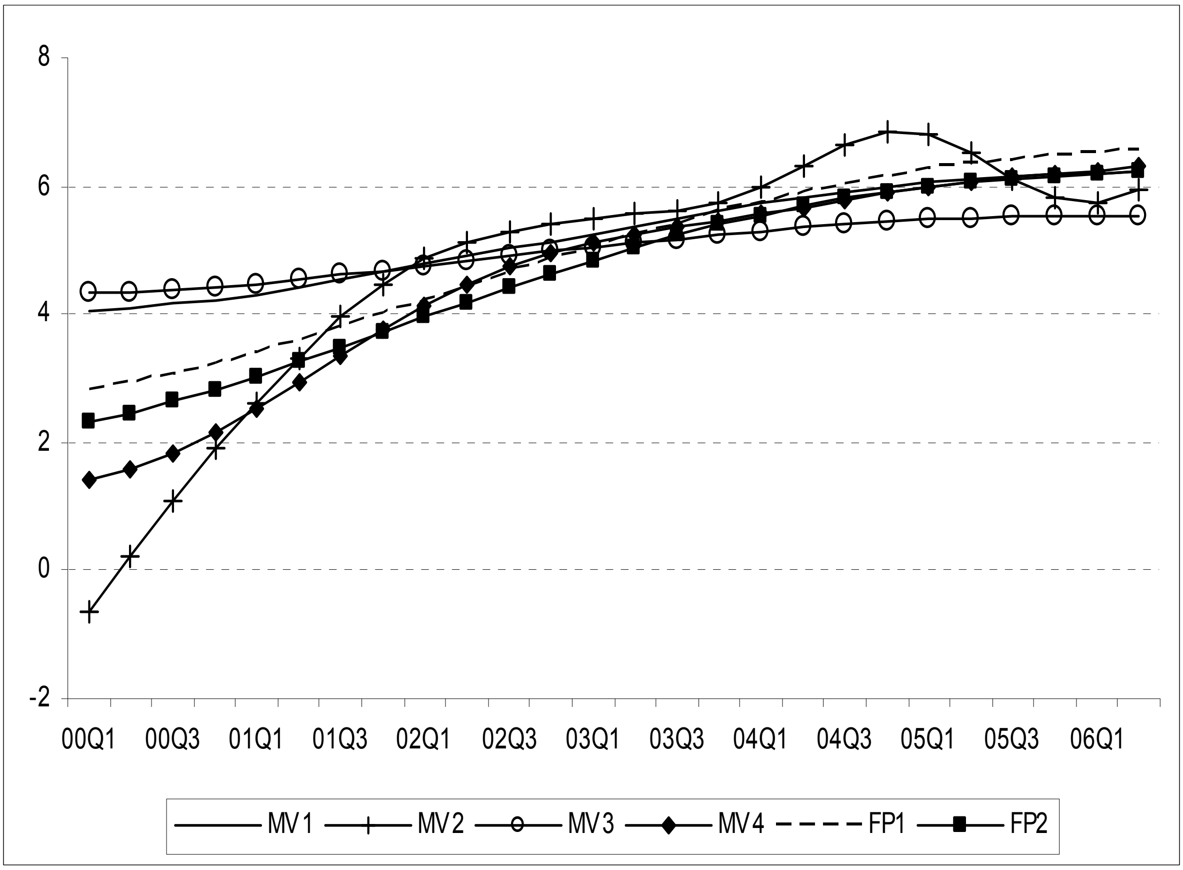
[8] See Appendix 2 for details on the Kalman filter.

The advantages of this approach are: the use of a large information set in the determination of potential GDP and excess demand; it is compatible with economic theory; the approach follows closely the definition of potential GDP as the level of GDP which does not generate inflationary pressures; the estimates regarding the current period are updated in the following quarters by incorporating new information. The disadvantage of the method is that it is relatively difficult to be implemented in practice since it requires several assumptions (for example, regarding the relative volatility of the trend and gap).

To estimate Romania's potential GDP we have used four versions of the multivariate filter with unobserved components. Details regarding the equations of each model and the results can be found in Appendix 5.

The average annual growth rate of potential GDP in the four versions of the multivariate filter for the 2003Q1 – 2006Q2 interval ranges between 5.4 and 6.1 percent (Table 1). The growth rates of potential GDP given by the four versions for the 1999Q4 – 2006Q2 period are presented in Figure 4.

Figure 4. Annual growth rates of potential GDP – multivariate filters (MV) and production function (PF) (%)

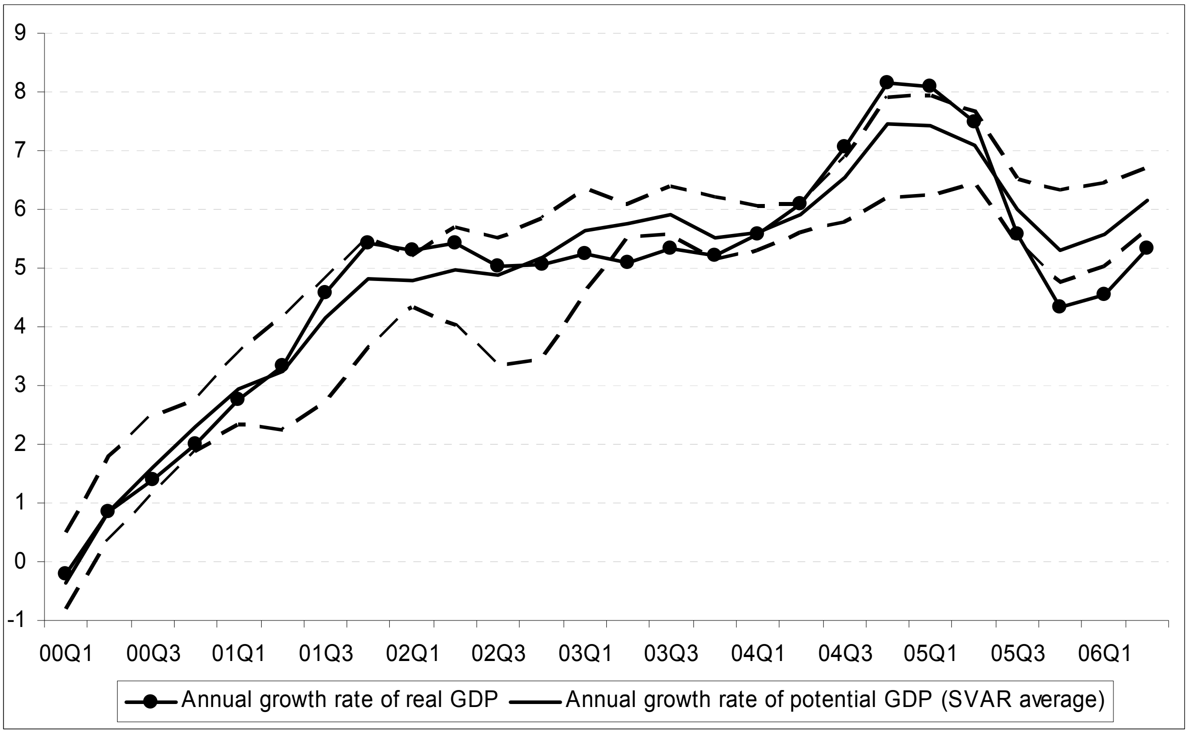


2.2.3. Structural vector autoregression (SVAR)

The quantification of the evolution of potential GDP through the structural vector autoregression (SVAR) method is based on estimating the relationship between real GDP growth and other macroeconomic variables in order to identify the permanent and temporary shocks to which the economy was subjected[9]. The main advantages of the method are: the possibility of a better economic interpretation of potential GDP variations; the gradual transmission of a permanent shock to potential GDP and the fact that all the results are based on econometric estimates, without the arbitrary setting of parameter values. Among the disadvantages are: higher data requirements compared to other methods and the risk of obtaining a potential GDP series which closely resembles the actual GDP series when the correlation between GDP and the other variables is weak.

SVARs with two and three variables were estimated for the period 1998Q1 – 2006Q2, including, alongside real GDP, other relevant macroeconomic variables: CPI inflation, CORE2 inflation, real gross wage, employment, the registered unemployment rate, industrial production and the real effective exchange rate.

Figure 5. Annual growth rates of potential GDP – SVAR (%)



Note: The growth rate of annual potential GDP is the mean of the growth rates resulted from the estimated SVAR models. The interval (with black dotted lines) is given by the minima, respectively maxima of the estimated growth rates of potential GDP.

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9 See Appendix 6 for details on using the SVAR model for measuring potential GDP.

The estimated SVARs indicate[10] an acceleration of the average annual growth rate of potential GDP from 3.3 percent in the 2000Q1 – 2002Q4 interval to 6.1 percent in 2003Q1 – 2006Q2. The values obtained from the SVAR models for the average annual growth rate of potential GDP in the 2003Q1 – 2006Q2 period range between 6 and 6.4 percent. The average of the potential GDP growth rates is presented in Figure 5; in order to assess the variability of the results between the SVAR models, the average growth rate is presented along with the minimum and maximum growth rate obtained for each period with different models. In periods in which the real GDP growth rate exceeds that of the potential GDP there is an increase in excess demand (a widening of the positive output gap or a closing of a negative gap), and in periods in which it is lower than the growth rate of potential GDP there is a decrease in excess demand.

3. CONCLUSIONS

This paper presents the results of estimating Romania's potential GDP growth rate with several different methods. These results should be interpreted with caution since they carry a high degree of uncertainty due to the unobservable nature of the measured variables and the rather small available data sample. In spite of this, the estimates arrived at with different methods and specifications offer similar conclusions, giving out a unitary message regarding the dynamics and approximate values of potential GDP growth rates in the period under scrutiny.

The results indicate an increasing annual potential GDP growth rate, from an average of 3-4 percent in the 2000-2002 period to values of around 6 percent in recent periods. Thus, Romania's potential GDP has grown at rates significantly above those registered by new EU Central and Eastern European member states and South European states in their periods of high growth;[11] the magnitude of Romania's growth rates is comparable to that of growth rates in the Baltic countries, being exceeded only by the size of growth rates estimated for Ireland and Latvia.

[10] The values presented are the average of the 9 estimated SVAR models. The results of each SVAR model are presented in Table 1.

[11] According to the results of Denis et al. (2006), most of these countries registered potential GDP growth rates ranging between 3 and 5 percent.

Table 1. Estimation results for the growth rate of annual potential GDP

Approach	Model	Lags	Average 2000Q1-2002Q4	Average 2003Q1-2006Q2	Average 2000Q1-2006Q2
SVAR	Registered unemployment rate	2	3.05	6.35	4.82
	CORE2 inflation	1	3.31	6.09	4.81
	CPI inflation	4	3.39	6.00	4.80
	Employment + CPI inflation	4	3.33	6.07	4.81
	Employment + CORE2 inflation	1	3.31	6.10	4.81
	Real effective exch. rate + CPI inflation	2	3.19	6.22	4.82
	Real effective exch. rate + CORE2 inflation	2	3.25	6.18	4.83
	Real gross wage + CORE2 inflation	4	3.36	6.10	4.83
	Industrial production index + CPI inflation	4	3.36	6.10	4.84
	Average		**3.28**	**6.13**	**4.82**
Production function	1st version		3.41	5.73	4.57
	2nd version		3.75	5.97	4.87
	Average		**3.58**	**5.85**	**4.72**
Univariate filters	Hodrick-Prescott		3.65	5.72	4.77
	Band-pass		2.98	6.24	4.73
	Model UV1		4.24	6.06	5.22
	Model UV2		3.23	5.94	4.69
	Average		**3.52**	**5.99**	**4.85**
Multivariate filters with unobserved components	Model MV1		4.52	5.86	5.25
	Model MV2		3.13	6.08	4.72
	Model MV3		4.61	5.37	5.02
	Model MV4		3.15	5.78	4.57
	Average		**3.85**	**5.77**	**4.89**

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Appendices

Appendix 1. Data sources[12]

Quarterly real GDP for the period 1998Q1-2006Q2 was computed starting from the constant price GDP (2003 average price) published by the National Institute of Statistics (NIS) for 2005, expanded based on the quarterly year-on-year growth rates of real GDP published in NIS bulletins starting from 1999Q1. Real gross fixed capital formation, used in constructing the fixed capital series, was computed in a way similar to the one described above for real GDP. Data on employment has been published by the NIS in "Labor Force in Romania: Employment and Unemployment" bulletin. Data regarding the industrial production index is taken from the "Industry Statistical Bulletin" of the NIS, data on the registered unemployment rate, the number of employees and the gross wage economy-wide from the "Monthly Statistical Bulletin" released by the NIS, while consumer price index (CPI) data from the "Prices Statistical Bulletin", published by the NIS. CORE2 inflation data (CPI excluding administered prices, fuel prices and volatile food prices)[13] are computed by the NBR based on the disaggregated CPI data provided by the NIS. Average RON/EUR and RON/USD exchange rate data, used in constructing the effective exchange rate are published in the NBR monthly bulletin, while the currency structure of foreign trade is taken from the National Customs Authority. Price indices for Euro Area (HICP) and USA (US CPI) are taken from ECB and the Bureau of Labor Statistics.

Real GDP and real gross fixed capital formation data series were seasonally adjusted using Demetra, while the unemployment rate, number of employees, gross wage and industrial production index data were seasonally adjusted using the X12-ARIMA procedure. The registered unemployment rate series spiked by more than 3 percentage points in January 2002, breaking the downward trend registered in the previous quarters[14]. The implementation of Law 416/2001 regarding the guaranteed minimum income generated this increase as suddenly a large number of people registered as unemployed although they were not fulfilling the necessary condition of actively searching for a job. In the following months, while these persons were eliminated from the statistics of the National Employment Agency , the above mentioned effect dissipated, the unemployment rate returning to its previous values by June 2002. Taking into account the artificial character and the high amplitude of this temporary increase, the unemployment rate data series was adjusted by eliminating the effect from the first and second quarters of 2002. Furthermore, the employment data series suffered major changes in the first quarter of 2002 owing to a change in NIS methodology. The series was adjusted by assuming zero growth between 2001Q4-2002Q1, and the data prior to 2001Q4 was recursively corrected using the quarterly differences taken from the data based on the previous methodology.

[12] Data used are available upon request (anca.galatescu@bnro.ro).
[13] See Box 1 from Inflation Report, August 2005, page 13.
[14] See Inflation Report, I/2002 (former series).

Appendix 2. The Kalman filter

Unobserved components models can be represented in state space form[15]. This representation assumes that the dynamics of a (p×1) vector y_t with observable variables can be described with the help of a (m×1) vector α_t of unobservable components. The state space representation is in the form of the following system:

$$\alpha_{t+1} = T_t\alpha_t + R_t\eta_t$$

$$y_t = Z_t\alpha_t + \varepsilon_t,$$

where T_t, Z_t and R_t are parameter matrices with dimensions (m×m), (p×m) and (m×r). The first equation is known as the state (transition) equation while the second one is the measurement (observation) equation. Vectors η_t and ε_t (dimensions (r×1) and (p×1)) are normally distributed η_t~N(0,Q_t), and ε_t~N(0,H_t), where Q_t and H_t are covariance matrices (dimensions (r×r) and (p×p)). It is assumed that the error vectors η_t and ε_t are uncorrelated for all lags, $E(\eta_t\varepsilon'_\tau)$=0 for any t and τ.

Let $a_{t+1/t}=E(\alpha_{t+1} \mid Y_t)$ and $P_{t+1}=Var(\alpha_{t+1} \mid Y_t)$, be the conditional mean and conditional variance of α_{t+1} using information up to moment t, where $Y_t=\{y_1, y_2,..., y_t\}$ represents the information set.

The recursive equations for determining the conditional mean and the conditional variance of the state variables are:

$a_{t+1}=T_ta_t+K_tv_t$ şi $P_{t+1}=T_tP_tL'_t+R_tQ_tR'_t$, for t=1,...,n, where

$K_t=T_tP_tZ'_tF_t^{-1}$, $\quad v_t=y_t$ - Z_ta_t,

$F_t=Z_tP_tZ'_T+H_t$, $\quad L_t=T_t$ - K_tZ_t.

This system of equations is known as the Kalman filter.

Let $a_{t/t}=E(\alpha_t \mid Y_t)$ and $P_t=Var(\alpha_t \mid Y_t)$ be the conditional mean and the conditional variance of α_t using information up to date t, where $Y_t=\{y_1, y_2,..., y_t\}$ is the information set up to the current moment. The contemporaneous filtration equations are given by:

$a_{t+1}=T_ta_{t/t}$ and $P_{t+1}=T_tP_{t/t}L'_t+R_tQ_tR'_t$, for t=1,...,n, where

$a_{t/t}=a_t+M_tF_t^{-1}v_t$, $\quad P_{t/t}=P_t$ - $M_tF_t^{-1}M'_t$ and

[15] Extensive details regarding the design and estimation of state space models and Kalman filter are presented in Hamilton (1994), chapter 13, or Harvey (2001).

$F_t = Z_t P_t Z_T' + H_t$, $\quad M_t = P_t Z_t'$, $\quad v_t = y_t - Z_t a_t$,

The parameters are estimated through maximum likelihood. The initial state vector α_1 is normally distributed $\alpha_1 \sim N(a_1, P_1)$ and assumed to be known. Taking logs of the likelihood function gives the following expression:

log $L(y) = -\frac{np}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{n}\left[\log|F_t| + v_t' F_t^{-1} v_t\right]$, where the F_t matrix is non singular.

v_t and F_t are computed with the help of the Kalman filter, using the equations above.

Appendix 3. Univariate filters

In estimating potential GDP, three types of univariate filters were used for the period 1998Q1 – 2006Q2.

The Hodrick-Prescott filter (HP – Hodrick and Prescott, 1997) is the most common univariate method used in the estimation of potential GDP in applied economic research. The filter determines the trend of a time series (potential level for the real GDP case) by simultaneously minimizing both the variance of the data around the trend (gap volatility) and the variance of the growth rate of the trend. The trade-off between the two generally conflicting objectives is solved through a multiplier, λ, setting their relative weights. For determining potential GDP, we used λ = 1600, which is the recommended value for quarterly data.

The band-pass filter (BP), introduced by Christiano and Fitzgerald (1995), decomposes the time series in components with periodical fluctuations, each of the components corresponding to a certain frequency/periodicity. For example, fluctuations occurring at a four-quarter period are identified with the seasonal components, while those occurring at a 6-32-quarter interval are usually associated with business cycles fluctuations. Potential GDP was computed by eliminating from the real GDP series the components with periodicity shorter than 32 quarters.

Each of the above-mentioned filters are particular cases of univariate unobserved components models, for which the Kalman filter is used to get the solution. The univariate unobserved components models decompose time series in a permanent component (trend), a cyclical component and a residual term, each having a law of motion. This structural decomposition is proposed by Harvey and Jaeger (1993), being used in restricted or unrestricted forms in other papers like Bjornland, Brubakk and Jore (2006), Cerra and Saxena (2000), Darvas and Vadas (2003), Faal (2005) and Guarda (2002).

The first univariate unobserved components model (Model UV1) decomposes the observable series (real GDP) in three components: potential GDP, excess demand and a residual term. Laws of motion are specified for each of these components. Namely, the growth rate of potential GDP is assumed to follow a random walk process, while for the excess demand (GDP gap) a second order autoregressive process is assumed. The random walk specification is preferred for the growth rate of potential GDP considering that data is available only for a short sample with important structural breaks. Model equations are presented below (standard errors in brackets):

$y_t = y_t^P + y\_gap_t + \varepsilon_t^y$, $\mathrm{Var}(\varepsilon_t^y)$=0.01

(1.75)

$y_t^P = y_{t-1}^P + \Delta y_t^P / 4$

$\Delta y_t^P = \Delta y_{t-1}^P + \varepsilon_t^P$, Var$(\varepsilon_t^P)$=0.83

(3.38)

$y\_gap_t = 0.81 y\_gap_{t-1} - 0.10 y\_gap_{t-2} + \varepsilon_t^C$, Var$(\varepsilon_t^C)$=2.85

(0.18) (0.11) (3.37)

where y_t is log of real GDP, y_t^P log of potential GDP, Δy_t^P is the growth rate of potential GDP and y_gap_t the GDP gap in percentage points. Error terms are identically and independently distributed (i.i.d.) and pairwise uncorrelated.

As in the first model, the second one (Model UV2) decomposes the real GDP series in three parts: potential GDP, excess demand and a residual term. While excess demand is still specified as a second order autoregression, in UV2 the level of potential GDP (as opposed to the growth rate in the previous model) follows a random walk process. The model's equations are presented below (standard errors in brackets), with variables having the same meaning as in the first model. Also in this case, error terms are i.i.d. and pairwise uncorrelated.

$y_t = y_t^P + y\_gap_t + \varepsilon_t^y$, Var$(\varepsilon_t^y)$=0.00

(0.29)

$y_t^P = y_{t-1}^P + \varepsilon_t^P$, Var$(\varepsilon_t^P)$=1.01

(0.22)

$y\_gap_t = 0.91 y\_gap_{t-1} - 0.40 y\_gap_{t-2} + \varepsilon_t^C$, Var$(\varepsilon_t^C)$=0.50

(0.41) (0.27) (0.50)

The annual growth rates of potential GDP resulted from applying the four univariate filters presented in this appendix are shown in Table 1. The average annual growth rate of potential GDP for the 2000Q1-2006Q2 period is 4.85%, while it is higher, namely 5.99% if only the period 2000Q1-2006Q2 is taken into account.

Appendix 4. Production function

For using the production function approach in estimating potential GDP, we used quarterly data for employment[16], GDP and gross fixed capital formation, the latter two in real terms, for the period 1998Q1-2006Q2. The types of production functions used in the literature are particular forms of the constant elasticity of substitution (CES) function, with the following general form in the case of two production factors[17]:

$$Y = A\left(\alpha K^{\gamma} + (1-\alpha)L^{\gamma}\right)^{1/\gamma} \qquad (*)$$

where:

Y is output, A is total factor productivity, K is the capital stock and L represents the labor force. Alpha (α) is the capital's contribution to output formation, while γ determines the degree of substitution between the two factors. Regarding the last parameter, the following relation is valid in connection with the elasticity of substitution: $\gamma = \frac{\sigma - 1}{\sigma}$.

In the particular case of unit elasticity of substitution, ($\sigma = 1$, equivalent with $\gamma = 0$), the relation (*) presented above becomes $Y = AK^{\alpha}L^{1-\alpha}$, namely a Cobb-Douglas production function.

Although the unit elasticity of substitution is a strong assumption, not imposing it would mean separately estimating its value or adopting a value used in similar studies, the latter approach being followed for example by Benk, Jakab and Vadas (2005) for Hungary[18]. Considering the lack of necessary data for estimating the elasticity of substitution parameter (usually a panel data approach is needed with detailed data for each firm), the lack of a study on Romania which could suggest values for the parameter and the large scale utilization in the literature of the Cobb-Douglas production function, this specification was also adopted for this paper.

Furthermore, trying to estimate capital and labor contributions to the output in the Cobb-Douglas production function did not yield economically meaningful results. On the other hand, small variations in these contributions (e.g. 0.7 for labor and 0.3 for capital) produce similar results. The assumed parameter values (0.67 for labor and 0.33 for capital) are also close to the ones of Dobrescu (2006), who finds a value of approximately 0.65 for the weight of labor in the production function.

The annual depreciation rate is assumed to be similar with the one used in most other studies, 5% (yielding a quarterly rate of 1.23%). A higher depreciation rate would be justified for the

[16] An average of 8 hours per day was taken into account for working hours.
[17] Although there can be other ways in which total factor productivity is incorporated.
[18] Even in this case the authors emphasize the high uncertainty degree of the estimated parameter.

beginning of the transition period, but taking into account the time elapsed and the presumably higher weight of buildings in the capital stock, the value mentioned above is a reasonable one.

The results are presented in Subsection 2.2.1 of the paper.

Appendix 5. Multivariate filters with unobserved components

The multivariate Kalman filter with unobserved components was used to estimate several models for the 1998Q1-2006Q2 period.

The first model (Model MV1) has two main equations: a Phillips curve and a law of motion for the output gap. Equations are presented below (standard errors in brackets).

$$\pi_t = 0.40\pi_{t-1} + (1-0.40)\pi_t^e + 0.69 y\_gap_t + \varepsilon_t^{\pi}, \ \mathrm{Var}\left(\varepsilon_t^{\pi}\right)=1.00 \quad (5.1)$$

(0.00) (0.01) (0.00)

$$y\_gap_t = 0.81 y\_gap_{t-1} - 0.10 y\_gap_{t-2} + \varepsilon_t^{ygap}, \ \mathrm{Var}\left(\varepsilon_t^{ygap}\right)=0.01 \quad (5.2)$$

(0.01) (0.01) (0.00)

$$\Delta y_t^P = \Delta y_{t-1}^P + \varepsilon_t^{yP}, \ \mathrm{Var}\left(\varepsilon_t^{yP}\right)=0.00 \quad (5.3)$$

(0.00)

$$y_t = y_t^P + y\_gap_t + \varepsilon_t^{y}, \ \mathrm{Var}\left(\varepsilon_t^{y}\right)=0.01 \quad (5.4)$$

(0.00)

π_t is CORE2 inflation and π_t^e represents a measure of inflation expectations. The inflation expectations were approximated by an equally weighted average of CPI inflation for the previous, current and following two quarters. As in the case of univariate filters, y_t is the log of real GDP, y_t^P log of potential GDP, Δy_t^P potential GDP growth rate and y_gap_t is the GDP gap in percentage points. Error terms are i.i.d. and pairwise uncorrelated.

Equation (5.1) shows the dynamics of current inflation as a function of past inflation, inflation expectations and excess demand. We impose the standard homogeneity restriction of the coefficients on past inflation and inflation expectations summing to 1. Equation (5.4) is an identity where the real GDP level is decomposed in a potential level, deviation from trend and a residual term. These components have the following laws of motion: a second order autoregressive process in the case of the deviation from trend (equation 5.2) and a random walk process for the growth rate of potential GDP (equation 5.3).

The average annual growth rate of potential GDP is 5.86% for the 2003Q1-2006Q2 period.

The simultaneous estimation of potential GDP and NAIRU

With the second multivariate model with unobserved components (model MV2) the potential GDP and the non-accelerating inflation rate of unemployment (NAIRU) are simultaneously identified. These types of models usually include an inflation equation, an Okun's law and equations describing the dynamics of potential GDP and NAIRU.

There are different approaches to the specification of the inflation equation, which depends on its own inertia, inflation expectations, excess demand and on various supply factors. Some authors, such as Apel and Jansson (1998), Cerra and Saxena (2000), Fabiani and Mestre (2001), Greenslade, Pierse and Salaheen (2003), estimate a version of the Phillips curve called the *triangle Phillips curve,* with a variable NAIRU, an approach first introduced by Gordon (1997). They use as dependent variable the first difference of the inflation rate, thus imposing a homogeneity condition, according to which, in the long run, there is no link between inflation and excess demand. Other authors, such as Benes and N'Diaye (2004), Bjornland, Brubakk and Jore (2006), use as dependent variable the level of the inflation rate and the homogeneity restriction is that the sum of coefficients on past inflation rates and on inflation expectations equal 1.

Likewise, there are different approaches regarding the laws of motion of NAIRU and growth rate of potential GDP. Namely, the assumed processes depend on the order of integration of the series and structural changes that took place in the economy throughout the analyzed period. Furthermore, Okun's law can be specified in a different manner depending on the causality relation between the cyclical components of unemployment and GDP. Therefore, depending on the characteristics of each economy, the authors consider as dependent variable in Okun's law either the output gap, or the deviation of the unemployment rate from its trend.

The estimated model has three equations: a Phillips curve, Okun's law and a law of motion for the deviation of the unemployment rate from its trend[19]. Standard errors are in brackets.

$$\pi_t = 0.24\pi_{t-1} + (1-0.24)\pi_t^e + 0.26 y\_gap_{t-1} + \varepsilon_t^{\pi}, \ \mathrm{Var}\left(\varepsilon_t^{\pi}\right)=5.77 \qquad (5.5)$$

(0.07) (0.04) (0.62)

$$y\_gap_t = -0.22 u\_gap_{t-1} - 0.45 u\_gap_{t-2} + \varepsilon_t^{ygap}, \ \mathrm{Var}\left(\varepsilon_t^{ygap}\right)=1.29 \qquad (5.6)$$

(0.65) (0.61) (0.71)

$$u\_gap_t = 0.85 u\_gap_{t-1} - 0.20 u\_gap_{t-2} + \varepsilon_t^{ugap}, \ \mathrm{Var}\left(\varepsilon_t^{ugap}\right)=0.35 \qquad (5.7)$$

(0.19) (0.23) (0.59)

[19] Two other versions of this model have been estimated. The first one reverses causality between the cyclical component of unemployment and GDP (equation 5.6), and the GDP gap appears contemporaneously in the inflation equation. For the period 2003Q1-2006Q2 the average growth rate of potential GDP is 6.01% per year. In the second version, the Phillips curve is specified having as dependent variable the first difference of CORE2 inflation, while the independent variables in the unemployment gap equation are the past values of the GDP gap. In this case, the average growth rate of potential GDP is 6.20% per year for the period 2003Q1-2006Q2.

$$\Delta y_t^P = \Delta y_{t-1}^P + \varepsilon_t^{yP}, \ \mathrm{Var}\left(\varepsilon_t^{yP}\right)=1.85 \quad (5.8)$$

(2.29)

$$u_t^* = u_{t-1}^* + \delta_t + \varepsilon_t^{u*}, \ \mathrm{Var}\left(\varepsilon_t^{u*}\right)=0.00 \quad (5.9)$$

(0.86)

$$\delta_t = \delta_{t-1} + \varepsilon_t^{\delta}, \ \mathrm{Var}\left(\varepsilon_t^{\delta}\right)=0.05 \quad (5.10)$$

(0.15)

$$y_t = y_t^P + y\_gap_t, \quad (5.11)$$

$$u_t = u_t^* + u\_gap_t, \quad (5.12)$$

Variables have the same interpretation as in the first model. Additionally, u_t is the registered unemployment rate, u_t^* is its trend and u_gap_t is the deviation from the trend. Also in this case, error terms are i.i.d. and pairwise uncorrelated.

Equation (5.5) shows the dynamics of the current inflation rate, as a function of past inflation, inflation expectations and excess demand. The coefficients on past inflation and on inflation expectations, respectively, sum to 1. This restriction is imposed because, in the long run, there is no link between the cyclical component of GDP and inflation.

Equation (5.6) represents "Okun's law", a relation between the unemployment rate gap and the output gap. Equation (5.7) is a second order autoregressive process for the unemployment gap. In addition, there is an equation for the growth rate of potential GDP, as a random walk process (5.8). The equations (5.9) and (5.10) describe a random walk with drift for the unemployment trend, δ_t (δ_t in turn, is a random walk process).

The average growth rate of potential GDP was 6.08 percent per year for the 2003Q1 – 2006Q2 period and 4.72 percent per year for the 2000Q1 – 2006Q2 period.

Simultaneous identification of the potential GDP and the neutral interest rate

The simultaneous identification of the natural interest rate (the neutral interest rate or the interest rate trend) and the growth rate of potential GDP using Kalman filter was introduced by Laubach and Williams (2001). In this context, the natural interest rate is defined as the level of the interest rate consistent with GDP staying at its potential level, respectively the level of GDP which does not put pressure on inflation. The same method was later also applied by Clark and Kozicki (2004), Crespo-Cuaresma, Gnan and Ritzberger-Gruenwald (2003), Garnier and Wilhelmsen (2005), Larsen and McKeown (2004), Llosa and Miller (2006), Manrique and Marques (2004).

Laubach and Williams (2001) specify the Phillips curve with core inflation as the dependent variable, as a function of its own lags, the output gap and the deviation of relative prices (such as import prices or the oil price) from core inflation. The aggregate demand equation includes lags of the output gap and of the real interest rate gap. The neutral interest rate is specified as a function of potential GDP growth and a term embedding other fundamental determinants of the neutral interest rate (such as the households' rate of time preference).

The third multivariate model with unobservable components (Model MV3) estimated in this paper consists of a Phillips curve, an equation for aggregate demand and laws of motion for the real interest rate gap, the neutral interest rate and the growth rate of potential GDP.

$$\pi_t = 0.48\pi_{t-1} + (1-0.48)\pi_t^e + 0.70 y\_gap_t + \varepsilon_t^{\pi},\ \text{Var}\left(\varepsilon_t^{\pi}\right)=20.53 \qquad (5.13)$$

(0.40) (0.63) (10.75)

$$y\_gap_t = 0.93 y\_gap_{t-1} - 0.70 y\_gap_{t-2} - 0.08 r\_gap_{t-1} - 0.18 r\_gap_{t-2} + \varepsilon_t^{ygap}$$

(1.09) (0.44) (0.27) (0.26)

$$\text{Var}\left(\varepsilon_t^{ygap}\right)=13.92 \qquad (5.14)$$

(26.93)

$$r\_gap_t = 0.55 r\_gap_{t-1} - 0.36 r\_gap_{t-2} + \varepsilon_t^{rgap},\ \text{Var}\left(\varepsilon_t^{rgap}\right)=29.39 \qquad (5.15)$$

(0.63) (0.57) (17.94)

The variables have the same significance as in the first model. Additionally, the interest rate considered in the model, r_t, is the average interest rate on NBR's sterilization operations (one-month deposit-taking operations, three-month certificates of deposit and the deposit facility). The real interest rate is computed *ex ante* using as an approximation for inflation expectations a simple arithmetic average of CPI inflation rates for the previous two quarters, the current and the following quarter. r_t^* stands for the neutral interest rate, and r_gap_t is the real interest rate gap. The error terms ε_t^{π}, ε_t^{ygap}, ε_t^{rgap}, ε_t^{yP}, ε_t^{y} and ε_t^{r*} are i.i.d. with no cross-correlation.

Equation (5.13) sets interest rate as a function of its persistence, inflation expectations and excess demand. In the aggregate demand equation (5.14) the output gap is a function of its own lags and the lags of the real interest rate gap. (5.15) is a second order autoregressive process for the real interest rate gap. (5.16) and (5.17) are identities for the components of GDP and the real interest rate. Equations (5.18) and (5.19) are laws of motion for the growth rate of potential GDP and the real interest rate trend.

$$y_t = y_t^P + y\_gap_t + \varepsilon_t^y \text{, Var}\left(\varepsilon_t^y\right)\text{=0.03} \quad (5.16)$$

(7.13)

$$r_t = r_t^* + r\_gap_t \text{,} \quad (5.17)$$

$$\Delta y_t^P = \Delta y_{t-1}^P + \varepsilon_t^{yP} \text{, Var}\left(\varepsilon_t^{yP}\right)\text{=0.07} \quad (5.18)$$

(0.82)

$$r_t^* = r_{t-1}^* + \varepsilon_t^{r*} \text{, Var}\left(\varepsilon_t^{r*}\right)\text{=3.68} \quad (5.19)$$

(4.15)

The results point to an average growth rate of potential GDP of 5.37 percent over 2003Q1-2006Q2.

The fourth multivariate model with unobservable components (Model MV4) has four main equations: a Phillips curve, an aggregate demand equation, a law of motion for the real interest rate gap and an equation for the deviation of the real exchange rate of the domestic currency from its trend.

$$\pi_t = 0.83 * (0.34\pi_{t-1} + (1-0.34)\pi_t^e) + 0.80 y\_gap_t + (1-0.83)(\pi\_pm_t - \Delta z_t^{tnd}) + \varepsilon_t^{\pi}$$

(0.06) (0.14) (0.10)

$$\text{Var}\left(\varepsilon_t^{\pi}\right)\text{=6.11} \quad (5.20)$$

(0.67)

$$y\_gap_t = 0.66 y\_gap_{t-1} - 0.10 r\_gap_{t-1} + 0.14 z\_gap_{t-1} + \varepsilon_t^{ygap} \text{,} \quad (5.21)$$

(0.05) (0.04) (0.04)

$$\text{Var}\left(\varepsilon_t^{ygap}\right)\text{=0.93}$$

(0.19)

The variables have the same significance as in the first model. Additionally, z_t stands for the logarithm of the effective real exchange rate, z_t^* for the logarithm of the trend of the real exchange rate, and z_gap_t for the real exchange rate gap. The average of the CPI inflation rates for the previous, current and the next two quarters was used as an approximation of inflation expectations. π_pm_t represents the inflation rate of import prices, computed using the HICP inflation rate and the nominal effective exchange rate. The error terms ε_t^{π}, ε_t^{ygap}, ε_t^{rgap}, ε_t^{zgap}, ε_t^{yP}, $\varepsilon_t^{r^*}$ and ε_t^{ztnd} are i.i.d. and are not cross-correlated.

Equation (5.20) describes the inflation rate as a function of its persistence, inflation expectations, output gap and import prices. In the aggregate demand equation (5.21) the output gap is a function of its own persistence and lags of the real interest rate and real effective exchange rate gaps. (5.22) – (5.24) are identities for the components of GDP, the real interest rate and the real effective exchange rate. Equations (5.25) – (5.27) are laws of motion for potential GDP growth, the real interest rate trend and the growth rate of the real effective exchange rate trend. Equations (5.28) and (5.29) are second order autoregressive processes for the real interest rate gap and the real effective exchange rate gap, respectively.

$$y_t = y_t^P + y\_gap_t, \tag{5.22}$$

$$r_t = r_t^* + r\_gap_t, \tag{5.23}$$

$$z_t = z_t^* + z\_gap_t, \tag{5.24}$$

$$\Delta y_t^P = \Delta y_{t-1}^P + \varepsilon_t^{yP}, \ \mathrm{Var}\left(\varepsilon_t^{yP}\right)=\underset{(0.04)}{0.10} \tag{5.25}$$

$$r_t^* = \underset{(0.00)}{0.95} r_{t-1}^* + \varepsilon_t^{r^*}, \ \mathrm{Var}\left(\varepsilon_t^{r^*}\right)=\underset{(0.00)}{0.04} \tag{5.26}$$

$$\Delta z_t^{tnd} = \Delta z_{t-1}^{tnd} + \varepsilon_t^{ztnd}, \ \mathrm{Var}\left(\varepsilon_t^{ztnd}\right)=\underset{(0.02)}{0.10} \tag{5.27}$$

$$r\_gap_t = \underset{(0.00)}{0.80} r\_gap_{t-1} - \underset{(0.00)}{0.35} r\_gap_{t-2} + \varepsilon_t^{rgap}, \ \mathrm{Var}\left(\varepsilon_t^{rgap}\right)=\underset{(0.00)}{0.30} \tag{5.28}$$

$$z\_gap_t = \underset{(0.06)}{0.76} z\_gap_{t-1} - \underset{(0.04)}{0.30} z\_gap_{t-2} + \varepsilon_t^{zgap}, \ \mathrm{Var}\left(\varepsilon_t^{zgap}\right)=\underset{(0.19)}{2.03} \tag{5.29}$$

Over 2003Q1-2006Q2, the average growth rate of potential GDP was 5.78 percent per year.

The estimation results for the four models are presented in Table 1.

Appendix 6. Structural vector autoregression (SVAR)

The evolution of potential GDP can be determined through structural vector autoregression (SVAR) models by imposing restrictions in the long run on the effect of structural shocks (restrictions of the type introduced by Blanchard and Quah, 1989). The measurement of the evolution of potential GDP by using the SVAR[20] method is done in three steps:

1. Estimation of a reduced VAR model

A reduced VAR model $A(L)x_t = u_t$ is estimated, where x_t is a vector of macroeconomic variables and u_t is a vector of i.i.d. residuals with zero mean and covariance matrix $E(u_t u'_t) = \Omega$. The x_t vector includes real GDP growth along with one or more other economic variables. The VAR model is often used to capture empirically the dynamic interdependency among macroeconomic variables without *a priori* imposing restrictions based on economic theory. If x_t is stationary, the Wold theorem implies that it can be represented as a moving average of past residuals $x_t = C(L)u_t$.

2. Identification of the structural shocks

The structural shocks which have affected the economy over the considered sample are identified by decomposing the residuals of the estimated VAR model. Thus, it is assumed that the reduced form residuals are linear combinations of the structural shocks $u_t = S\varepsilon_t$, where the structural shocks are contemporaneously uncorrelated $E(\varepsilon_t \varepsilon'_t) = I_n$. With the notation $D(L) = C(L)S$ the model is rewritten in the structural VAR (SVAR) form as $x_t = D(L)\varepsilon_t$, explaining the dynamics of the economic variables as driven by past structural shocks.

To decompose the residuals we need some restrictions to allow the unique identification of S, the matrix which controls the way structural shocks combine to generate the estimated residuals u_t. In order to measure the growth rate of potential GDP, the restrictions are of the Blanchard-Quah type. These are expressed in the condition that $D(1) = C(1)S$ is an inferior triangular matrix, practically requiring that the cumulated effects of the ε_{it} structural shock (the *i*-th shock in the model) on each of the x_{jt} variables with j<i be null (e.g., the cumulated effects of the third structural shock on the first two variables would be null). Consequently, the ordering of the variables in the VAR model matters for the structural decomposition. The conditions underlying the Blanchard-Quah restrictions are suggested by the requirement that the variation of the output gap becomes zero in the long run. The structural decomposition of the residuals was computed using the algorithm proposed by Hoffman (2001).

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[20] Using structural VAR models is part of the standard econometric methodology, so detailed explanations can be found in most time series textbooks (e.g. Hamilton, 1994, or Lutkepohl, 2005).

3. Determination of the growth rate of potential GDP

In the analysis, the first variable in the VAR is the quarterly growth rate of real GDP. As a result of the restrictions, the first structural shock has permanent effects on economic growth, while the other structural shocks identified have only transitory effects (the long-run cumulated effect each of these has on the growth rate of real GDP is zero). The growth rate of potential GDP is identified with the component of real GDP growth determined by permanent shocks, while the output gap variation is identified with the component determined by the transitory shocks.

As the methodology achieves a decomposition of economic growth in potential GDP growth and the variation of the output gap, to measure the level of potential GDP and the output gap it is necessary to additionally determine/impose an initial condition regarding the value of excess demand at one moment in the sample. Considering that such an initial condition would be arbitrary, as well as that the intention of the paper is to measure the growth rate of potential GDP, we do not use the estimation results to measure the level of the output gap.

To measure potential GDP growth we estimated SVAR models over the 1998Q1-2006Q2 sample. The small data sample available puts restrictions on the number of variables considered in the same model, as introducing additional variables leads to a rapid loss of degrees of freedom through the large number of parameters to be estimated. The importance of this restriction is increased by the fact that introducing a large number of variables with a small sample may force the selection of insufficient lags, thus leading to systematic errors in the estimation of the structural components (DeSerres and Guay, 1995). Consequently, the estimated SVARs include two or three variables – the growth rate of real GDP and one or two explanatory factors.

The explanatory variables are chosen to reflect the effect of various types of transitory shocks: temporary demand shocks (CPI inflation, CORE2 inflation), supply shocks (real gross wage, the number of employees, registered unemployment rate, the volume of industrial production) or shocks on the effective real exchange rate (which could be the effects of monetary policy shocks or external shocks). In the SVARs with three variables, one of the explanatory variables is always CPI or CORE2 inflation in order to catch the effect of temporary demand shocks (considered relevant for monetary policy purposes). In these cases, the inflation rate appears last in the VAR to allow the demand shock to produce temporary effects on all the other variables.

All the variables have been checked for unit roots using the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests. For the nonstationary variables (real GDP, the number of employees, the real effective exchange rate, the real gross wage, and the industrial production index) we used the first difference in estimation so that the VAR models would be stable. In some cases (CPI inflation, the unemployment rate), the two tests have conflicting results; as the estimated VARs have all the characteristic roots inside the unit circle, the variables were considered stationary. The optimal lag length was chosen based on the Akaike information criterion to insure a number of lags large enough to avoid systematic errors; the lag length thus chosen is in all cases greater or equal to the one suggested by the Wald and LR tests, which give good results regarding the minimum number of necessary lags according to DeSerres and Guay (1995).

The signs of the impulse response functions (computed based on the identified structural shocks) confirm the interpretation of shocks, generally being in line with the intuition suggested by economic theory[21]. Thus, a permanent shock on real GDP (considered a potential GDP shock) leads in the estimations to a decrease in the inflation rate (both CPI and CORE2), to a permanent decrease in the rate of unemployment and to accelerated growth in the number of employees, permanent increase in the industrial production, intensification of the real gross wage growth and of exchange rate appreciation. A positive demand shock is reflected in a temporary rise in excess demand, paired by temporary accelerated growth in the number of employees/ temporary decrease in the unemployment rate, as well as temporary intensification in the growth of industrial production and of the real wage. A positive supply shock (permanently higher growth of the industrial production or the number of employees/ permanent drop in the unemployment rate) leads to a temporary decrease of the output gap and a decrease in inflation. A cost-push shock, by permanently raising the real gross wage, leads to higher inflation and to transitory oscillations in economic growth, while a depreciation shock feeds into a temporary increase in excess demand and higher inflation.

The variance decomposition shows that the explanatory variables included in the estimated VAR models explain in most cases less than 30 percent of the variation in real GDP growth (Table 2). As such, the reduced explanatory power may lead to underestimation of the effect of the transitory structural shocks (mainly illustrated by these explanatory variables) on economic growth. An underestimation of the variation of excess demand (defined as the cumulated impact of transitory shocks) implies, in counterpart, an overestimation of the variation in potential GDP. Thus, the estimated potential GDP growth follows too closely the evolution of actual real GDP or, put another way, the method produces growth rates of potential GDP which are too high in periods of rapid growth and too low in periods of slow growth (effect visible in Figure 5). From this point of view, we favor the three-variable SVAR models which, due to the additional variable, explain the evolution of real GDP better than the two-variable ones. This effect is illustrated by the results for the average annual growth rate of potential GDP over 2003Q1-2006Q2, which stays in a tighter range in the case of trivariate SVARs (6.1-6.2 percent) than in the case of bivariate SVARs (6.0-6.4 percent). On the other hand, the trivariate

[21] These are only general, qualitative comparisons, as it is difficult to compare the results of estimated VARs with specific theoretical models as a result of the restrictions implicit in the specification; thus, for instance, in the specifications above it is impossible to identify a supply shock producing temporary effects on GDP as well as on the rate of unemployment.

SVAR models are generally based on a larger number of estimated parameters using the same data sample, so the degree of uncertainty in the estimation is higher.

Table 2. The explanatory power of the variables for the growth rate of real GDP

SVAR model	Lags	R^2 in the real GDP growth equation (%)	The weight of the effect of transitory shocks in the variance decomposition of the real GDP growth (%)	
			1 quarter	10 quarters
Registered unemployment rate	2	24.2	20.5	18.1
CORE2 inflation	1	13.5	4.4	4.4
CPI inflation	4	29.9	11.9	13.4
No. employees + CPI inflation	4	43.6	13.9	28.5
No. employees + CORE2 inflation	1	15.0	5.9	5.8
Real effective exch. rate + CPI inflation	2	24.7	29.5	24.6
Real eff. exch. rate + CORE2 inflation	2	26.3	30.2	26.1
Real gross wage + CORE2 inflation	4	49.0	14.1	35.0
Industrial prod. index + CPI inflation	4	47.9	45.6	58.0

Observation: the variance decomposition is based on the structural decomposition used for computing the growth rate of potential GDP.

The estimation results regarding the growth rate of potential GDP are presented in Table 1. Each line in the table represents an estimated model including quarterly real GDP growth and the explanatory variable or variables in the first column. The results point to the acceleration of average annual potential GDP growth from 3.3 percent over 2000Q1-2002Q4 to 6.1 percent over 2003Q1-2006Q2. The results of the SVAR models place the average annual growth rate of potential GDP over 2003Q1-2006Q2 in the range of 6.0-6.4 percent.